Firms' Goals and Profit Maximization

If the manager of a firm is to pursue the goal of profit maximization, he or she must, by definition, make the difference between the firm's revenue and its total costs as large as possible. In making such calculations, it is important that the manager use the economist's notion of costs—that is, the cost figure should include allowances for all opportunity costs.

إذا كان مدير الشركة يهدف إلى تحقيق أكبر ربح ممكن للشركة ، فيجب عليه ، أن يجعل الفرق بين إيرادات الشركة وإجمالي تكاليفها بأكبر قدر ممكن. عند إجراء مثل هذه الحسابات ، من المهم أن يستخدم المدير فكرة الاقتصاديين عن التكاليف - أي أن قيمة التكاليف يجب أن يشمل جميع التكاليف الفعلية و تكاليف الفرصة البديلة.

Marginalism

If managers are profit maximizers, they will make decisions in a marginal way. They will adjust the things that can be controlled until it is impossible to increase profits further. The manager looks, for example, at the incremental (or marginal) profit from producing one more unit of output or the additional profit from hiring one more employee.

Marginal profit: the additional profit from producing one more unit of output.

The Output Decision

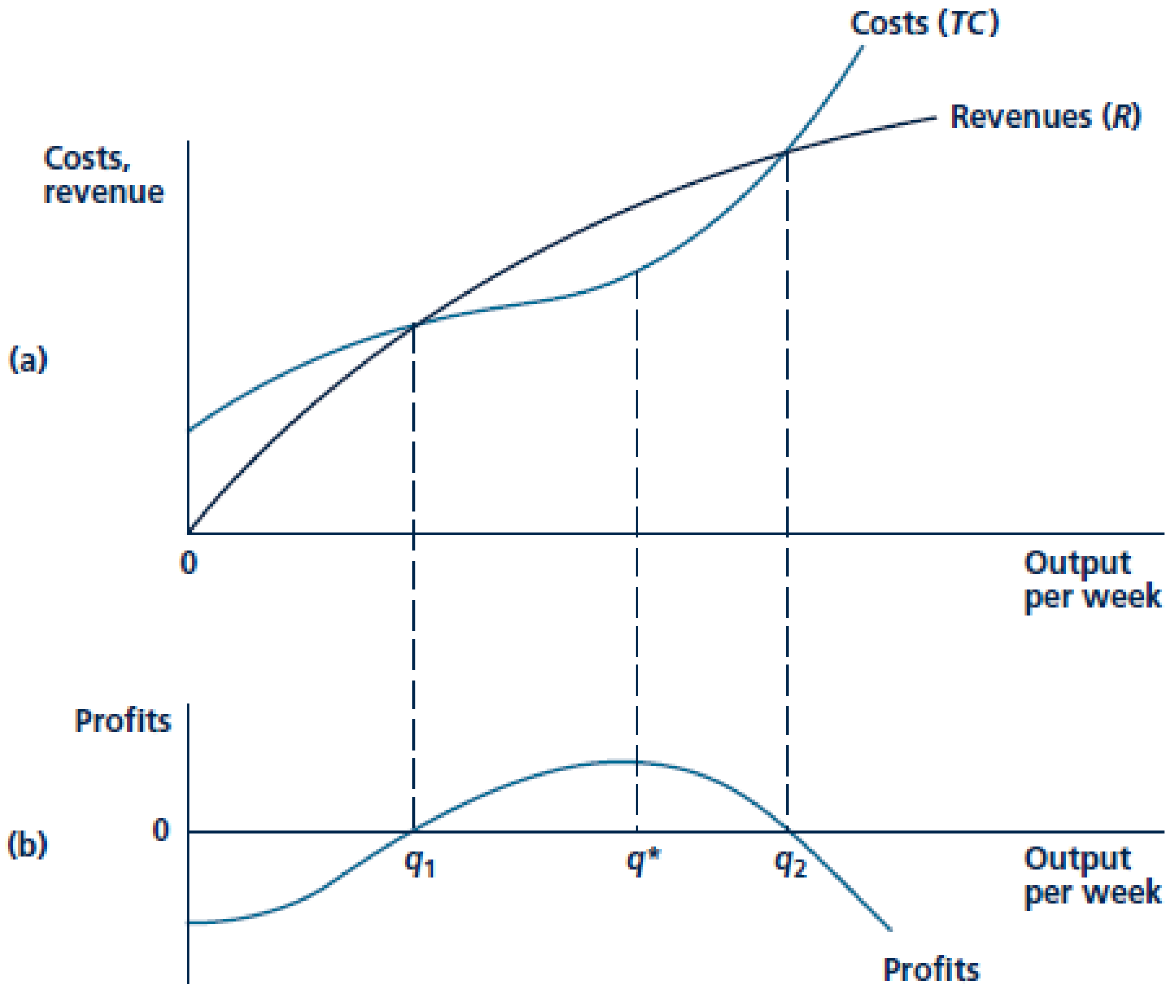
We can show this relationship between profit maximization and marginalism most directly by looking at the output level that a firm chooses to produce. A firm sells some level of output, q, and from these sales the firm receives its revenues, $\mathrm{R(q)}$. The amount of revenues received obviously depends on how much output is sold and on what price it is sold for. Similarly, in producing q, certain economic costs are incurred, $\mathrm{TC(q)}$, and these also depend on how much is produced. Economic profits (π) are defined as

$$\pi(q) = R(q) - TC(q)$$

Notice that the level of profits depends on how much is produced. In deciding what output should be, the manager chooses that level for which economic profits are as large as possible.

This process is illustrated in Figure below. The top panel of this figure shows rather general revenue and total cost curves. As might be expected, both have positive slopes—producing more causes both the firm's revenues and its costs to increase. For any level of output, the firm's profits are shown by the vertical distance

between these two curves. These are shown separately in the lower panel of Figure. Notice that profits are initially negative. At an output of q = 0 the firm obtains no revenue but must pay fixed costs (if there are any). Profits then increase as some output is produced and sold. Profits reach zero at q1—at that output level revenues and costs are equal. Beyond q1, profits increase, reaching their highest level at q*. At this level of output, the revenue and cost curves are furthest apart. Increasing output even beyond q* would reduce total profits–in fact, in this case, increasing output enough (to more than q2) would eventually result in profits becoming negative. Hence, just eyeballing the graph suggests that a manager who pursues the goal of profit maximization would opt to produce output level q*.



The Marginal Revenue/Marginal Cost Rule

Marginal revenue: The extra revenue a firm receives when it sells one more unit of output.

At output levels below q* increasing output causes profits to increase (MR > MC), so profit maximizing firms would not stop short of q*.

Increasing output beyond q* reduces profits (MC > MR), so profit maximizing firms would not produce more than q*.

At q* marginal cost equals marginal revenue, the extra revenue a firm receives when it sells one more unit of output.

In order to maximize profits, a firm should produce that output level for which the marginal revenue from selling one more unit of output is exactly equal to the marginal cost of producing that unit of output.

If at output level where MR > MC, the firm can increase its profit by increasing production.

If at output level where MC > MR, the firm can increase its profit by decreasing production.

If at output level where MC = MR, the firm maximizing profit.

We conclude that profit is maximized when $MR(q) = MC(q)$

Profit, $\pi = R - C$, is maximize at the point at which an additional increment to output leaves profit unchanged (marginal profit =0).

$$\pi = R - C \quad \Rightarrow \quad \frac{\Delta\pi}{\Delta q} = \frac{\Delta R}{\Delta q} - \frac{\Delta C}{\Delta q} = 0$$

$\frac{\Delta R}{\Delta q}$ = marginal revenue ; $\frac{\Delta C}{\Delta q}$ = marginal cost

At profit maximization $MR = MC$ ⇨ marginal profit = 0

<u>**Example**</u>

A firm produce good Z has a short run total cost function is $TC = 0.1q^2 + 10q + 200$. And total revenue function is $TR = 50\ q$.

a. What output level should the firm produce to maximum profits?

   To max profit: *MR = MC*

   $MC = 0.2q + 10$

   $MR = 50$

   $MR = MC \Rightarrow 0.2q + 10 = 50 \Rightarrow 0.2q = 40 \Rightarrow q = 40/0.2 = 200$ units

b. What is the firm's profit at this output level:

   $\pi = TR - TC = 50\ q - 0.1q^2 - 10q - 200 \rightarrow \pi = 40q - 0.1q^2 - 200$

   $\pi\ (\text{When } q = 200) = 40(200) - 0.1(200)^2 - 20 = 8{,}000 - 4{,}000 - 200 = 3{,}800.$

Marginal Revenue

If a firm can sell all it wishes without affecting market price—that is, if the firm is a price taker– the market price will indeed be the extra revenue obtained from selling one more unit. In other words, *if a firm's output decisions do not affect market price, marginal revenue is equal to price*. Suppose a firm was selling 50 widgets at $1 each. Then total revenues would be $50. If selling one more widget does not affect price, that additional widget will also bring in $1 and total revenue will rise to $51. Marginal revenue from the 51st widget will be $1 ($51 _ $50). For a firm whose output decisions do not affect market price, we therefore have : $MR = P$

For a price-taking firm: $MR = P$

Price taker: A firm or individual whose decisions regarding buying or selling have no effect on the prevailing market price of a good.

Because it is a price taker, the demand curve facing an individual competitive firm is given by a horizontal line. The demand curve facing the firm is horizontal because the firm's sales will have no effect on price.

Profit maximization by a Competitive Firm:

The general rule from profit maximization that is $MR = MC$. Because the demand curve facing a competitive firm is horizontal, so that $MR = P$. a perfectly competitive firm should choose its output so that marginal cost equal price.

$MC = MR = P$

Example

Suppose a firm's short run cost curve: $STC = 1 + 2q + q^2$. Assume the firm behaves as a price taker and sell its output at P = $50 per unit, If the firm maximizes profits, how much will it produce?

To max profit: $MC = P$

$2 + 2q = 50 \Rightarrow q = 48/2 = 24$ units.

Example

A firm uses a single input to produce its output. The production function is given by $q = 4L^{0.5}$. If the price of the product produced is $60 per unit and the cost of the labor input is $10, how much profit will the firm make if it maximizes profit?

$TC = w L = 10 L$ but $q = 4L^{0.5} \Rightarrow q/4 = L^{0.5} \Rightarrow$ (بتربيع الطرفين) $q^2 / 16 = L$

$\Rightarrow TC = 10 (q^2 / 16) = (10/16) q^2$

To max profit: $MC = P \Rightarrow (20/16) q = 60 \Rightarrow q = 48$ *units*

$\pi = TR - TC = P*q - TC = 60q - (10/16)\, q^2$

$\pi = 60(48) - (10/16)\,(48)^2 = 2{,}880 - 1{,}440 = \$1{,}440$

Example

Suppose you are the manager of a shoes producing firm operating in a competitive market. Your cost of producing is given by: $VC = 5q + 2q^2$, where q is the level of output produced.

a. if the market price of a pair of shoes is \$65, how many pairs of shoes would your firm produce?

To max profit in a competitive market: $MR = MC = P$

$$MC = \frac{\partial TC}{\partial q} = \frac{\partial VC}{\partial q} = 5 + 4q$$

⇨ $5 + 4q = 65$ ⇨ $4q = 60$ ⇨ $q = 15$ *units*

b. Now if the firm's fixed cost is known to be \$15, what is the profit for the firm?

Total cost = variable cost + fixed cost

$TC = 5q + 2q^2 + 15$

$\pi = TR - TC = P*q - TC = 65q - 5q - 2q^2 - 15 = 60q - 2q^2 - 15$

$\pi = 60(15) - 2(15)^2 - 15 = 900 - 450 - 15 = \$435$

Marginal Revenue for a Downward-Sloping Demand Curve

A firm that is not a price taker faces a downward sloping demand curve for its product.

These firms must reduce their selling price in order to sell more goods or services. In this case marginal revenue is less than market price $(MR < P)$.

A Numerical Example

Assume the quantity demanded of tape cassettes from a particular store per week (q) is related to the price (P) by $q = 10 - P$.

Total revenue is (P*q) and marginal revenue (MR) is the change in total revenue due to a change in quantity demanded.

- This example demonstrates that $MR < P$ as shown in Table.
- Total revenue reaches a maximum at $q = 5$, $P = 5$.

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- For $q > 5$, total revenues decline causing marginal revenue to be negative.

PRICE (P)	QUANTITY (Q)	TOTAL REVENUE (P · Q)	MARGINAL REVENUE (MR)
$10	0	$ 0	
9	1	9	$ 9
8	2	16	7
7	3	21	5
6	4	24	3
5	5	25	1
4	6	24	–1
3	7	21	–3
2	8	16	–5
1	9	9	–7
0	10	0	–9

Example

A firm faces a demand curve given by: $q = 100 - 0.2P$.
The firm has total cost curve of the form $TC = 2q^2 + 24q + 30$.

a. Calculate the marginal revenue curve in term of q.

$$TR = P * q$$

From the demand function $P = 500 - 5q$

$$TR = (500 - 5q)\ q = 500q - 5q^2.$$

$$MR = \frac{\partial TR}{\partial q} = 500 - 10q$$

b. What output level should the firm produce to maximize profit?

To max profit: $MR = MC$

$$MC = \frac{\partial TC}{\partial q} = 4q + 24$$

⇨ $4q + 24 = 500 - 10q$ ⇨ $14q = 476$ ⇨ $q = 476/14 = 34$ units.

Example:

If the demand curves a firm face is $P = 100 - 2q$ and a constant marginal and average cost of $16. What is the firm maximum profit?

$$TR = P * q$$

From the demand function $P = 100 - 2q$

$$TR = (100 - 2q)\, q = 100q - 2q^2.$$

$$MR = \frac{\partial TR}{\partial q} = 100 - 4q.$$

To max profit: $MR = MC \Rightarrow 100 - 4q = 16 \Rightarrow 4q = 84 \Rightarrow q = 21\, units$

$$\pi = TR - TC = TR - (AC * q) = 100q - 2q^2 - 16q = 84q - 2q^2$$

Example

Suppose a firm faces the following demand curve: $q = 60 - 2P.$

a. Calculate the total revenue curve for the firm in terms of q.

$$TR = P*q$$

From the demand curve: $P = \frac{60-q}{2} \Rightarrow P = 30 - ½q$

$$TR = (30 - ½q)\, q = 30q - ½q^2$$

b. Show that the firm's MR curve is given by: $MR = 30 - q$.

$$MR = \frac{\partial TR}{\partial q} = 30 - 2(½q) = 30 - q .$$

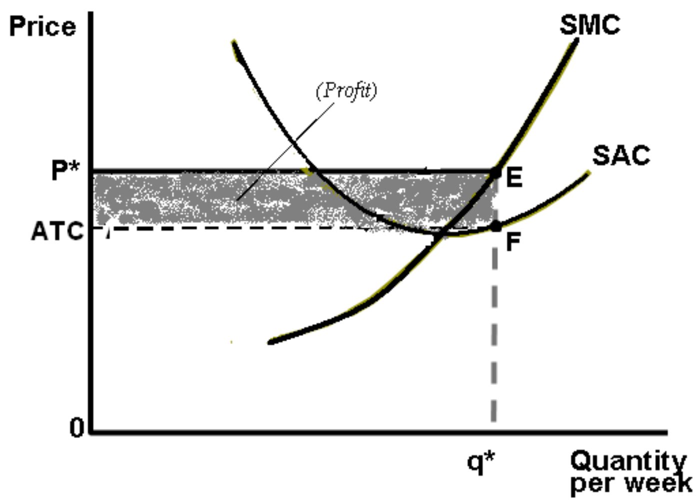
Short Run Profit maximization by a Competitive Firm: Graphical Analysis

Since the firm has no effect on the price it receives for its product, the goal of maximizing profits dictates that it should produce the quantity for which marginal cost equals price.

- The firm maximizes profits by producing q*, since this is where price equals short-run marginal costs.
- At P* profits are positive since $P > SAC$.
- If price just equaled average cost (and marginal cost), short-run profits equal zero.

- If, at P* the firm produced less than q*, profits could be increased by producing more since MR > SMC below q*. Alternatively, if the firm produced more than q* profits could be increased by producing less since MR < SMC beyond q*.

- Total profits are given by the area P*E F ATC which can be calculated by multiplying profits per unit (P* - ATC) times the firm's chosen output level q*.



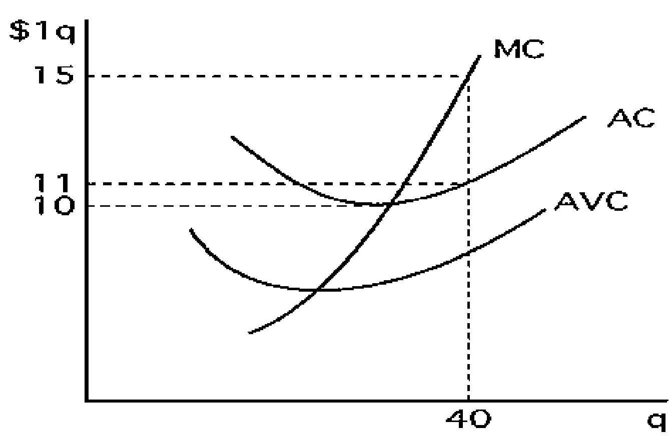
*Profit = total revenue – total cost. = P*q –TC*

We can factor q out of this expression to get: $profit = q\,(P - \frac{TC}{q}) = q\,(p - ATC)$.

Profit $(\pi) = q\,(p - ATC)$.

Example

The Figure shows the cost curves for a competitive firm. If the market price is $15 per unit. What is the firm profit?



$Profit = q\,(P - ATC) = 40\,(15 - 11) = 40 \times 4 = \$160$

Marginal Revenue and Price Elasticity

We define the price elasticity of demand for a single firm's output (q) as: $e_{Q,P} = \frac{Percentage\ change\ in\ q}{Percentage\ change\ in\ P}$

The relationship between elasticity and total expenditures (TR)

- If demand facing the firm is inelastic ($0 < e_{Q,P} < 1$), a rise in price will cause total revenues to rise .
- If demand is elastic ($e_{Q,P} > 1$), a rise in price will result in smaller total revenues (P↑ ⇨ TR ↓).
- If demand is unit elastic ($e_{Q,P} = 1$), a change in price will result in no change in total revenues .

The connection between the price elasticity of the demand and marginal revenue.

When demand is elastic ($e_{Q,P} > 1$), a fall in price raises quantity sold to such an extent that total revenues rise. Hence, in this case, an increase in quantity sold lowers price and thereby raises total revenue—marginal revenue is positive (MR > 0).

When demand is inelastic ($e_{Q,P} < 1$), a fall in price, although it allows a greater quantity to be sold, reduces total revenue. Since an increase in output causes price and total revenue to decline, MR is negative.

If demand is unit elastic ($e_{Q,P} = 1$), total revenue remains constant for movements along the demand curve, so MR is zero.

More generally, the precise relation between MR and price elasticity is given by:

$$MR = P\left(1 - \frac{1}{e_{Q,P}}\right)$$

If demand is elastic, Equation above shows that MR is positive. Indeed, if demand is infinitely elastic ($e_{Q,P} = \infty$), MR will equal price since, as we showed before, the firm is a price taker and cannot affect the price it receives.

TABLE 8.2 Relationship between Marginal Revenue and Elasticity

DEMAND CURVE	MARGINAL REVENUE
Elastic ($e_{q,P} > 1$)	$MR > 0$
Unit elastic ($e_{q,P} = 1$)	$MR = 0$
Inelastic ($e_{q,P} < 1$)	$MR < 0$

Example

Suppose that a firm faces a demand curve that has a constant elasticity of -2 . This demand curve is given by: $q = \frac{256}{P^2}$. Suppose also that the firm has a marginal cost curve of the form: $MC = 0.001q$.

a. Calculate the marginal revenue curve associated with the demand curve.

$$MR = P\ (1 + \frac{1}{e_{Q,P}})$$

$$MR = P\ (1 + \frac{1}{-2}) \quad \rightarrow \quad MR = ½\, P$$

From demand curve: $P^2 = \frac{256}{q} \quad \rightarrow \quad P = \frac{\sqrt{256}}{\sqrt{q}} \quad \rightarrow P = \frac{16}{\sqrt{q}}$

$$MR = ½\left(\frac{16}{\sqrt{q}}\right)$$

b. What output level should the firm produce to maximize profit?

To max profit: MR = MC $\rightarrow \frac{16}{2\sqrt{q}} = 0.001q \rightarrow 0.002q^{3/2} = 16 \rightarrow q^{3/2} = 8{,}000 \rightarrow q = (8{,}000)^{2/3} =$ 400 units

Example

Assume that a firm's marginal cost is \$10 and the elasticity of demand is -2. What is the firm's profit maximizing price?

$$MR = P\ (1 + \frac{1}{e_{Q,P}})$$

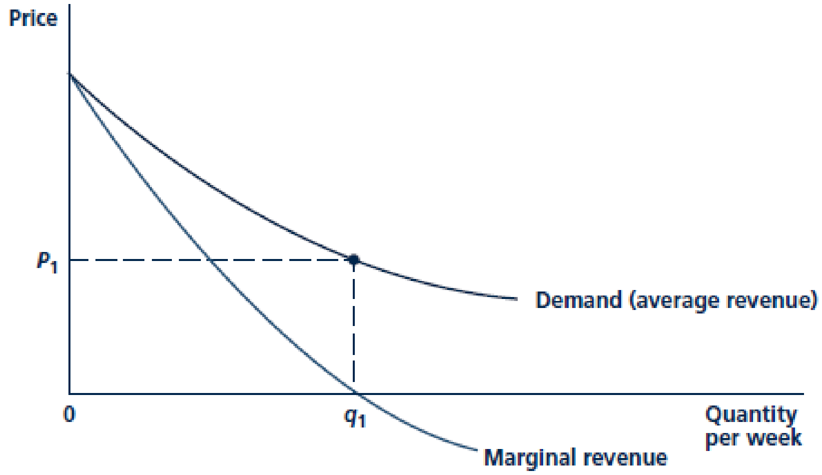
Firm maximize profit $\rightarrow \quad MR = MC = 10$

$$10 = P(\ 1 + -½) \quad \rightarrow \quad 10 = ½P \quad \rightarrow \quad P = \$20$$

Marginal revenue curve

A curve showing the relation between the quantity a firm sells and the revenue yielded by the last unit sold. Derived from the demand curve.

In the usual case of a downward-sloping curve, the marginal revenue curve will lie below the demand curve because, at any level of output, marginal revenue is less than price. In Figure below, we have drawn a marginal revenue curve together with the demand curve from which it was derived. For output levels greater than q_1, marginal revenue is negative. As q increases from 0 to q_1, total revenues (P*q) increase. However, at q_1, total revenues (P_1*q_1) are as large as possible; beyond this output level, price falls proportionately faster than output rises, so total revenues fall.



Example

Suppose a firm faces the following demand curve: $q = 60 - 2P.$

c. Calculate the total revenue curve for the firm in terms of q.

TR = P*q

From the demand curve: P = (60 - q) /2 → P = 30 – ½q

TR = (30 – ½q) q = 30q – ½q²

d. Show that the firm's MR curve is given by: MR = 30 - q .

MR = $\frac{dTR}{dq}$ = 30 - 2(½q) = 30 - q .

When Should the Firm Shut Down?

Suppose a firm is losing money. Should it shut down and leave the industry? The answer depends in part on the firm's expectations about its future business conditions. If it believes that conditions will improve and the business will be profitable in the future, it might make sense to operate at a loss in the short run. But let's assume for the moment that the firm expects the price of is product to remain the same for the foreseeable future. What, then, should it do?

Note that the firm is losing money when its price is less than average total cost at the profit-maximizing output q. In that case, if there is little chance that conditions will improve, it should shut down and leave the industry. This decision is appropriate even if price is greater than average variable cost.*

The Shutdown Decision*:*

The firm will produce when: TR > SVC

Dividing by q ⇨ $\frac{TR}{q} > \frac{SVC}{q}$ ⇨ $\frac{P \times q}{q} > \frac{SVC}{q}$ ⇨ P > SAVC

The firm will shut down when:

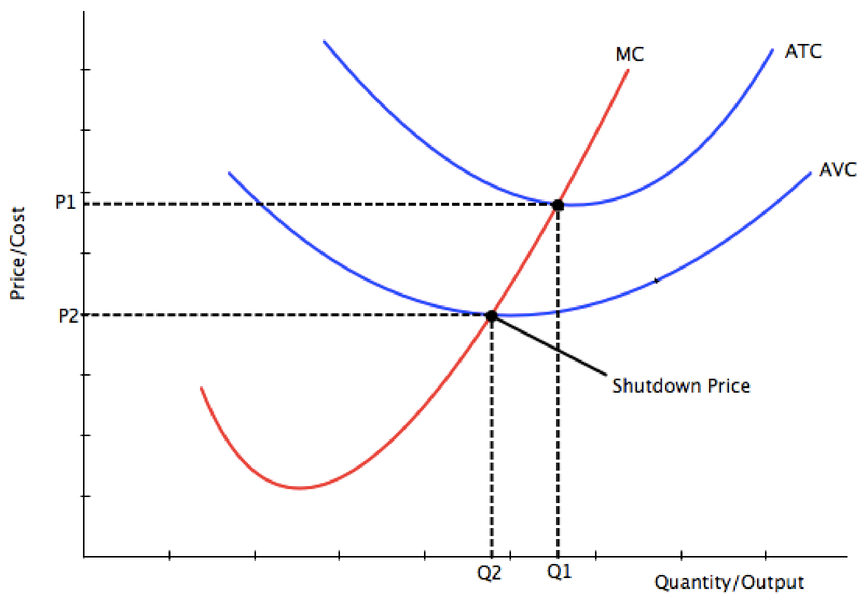
Total revenue < short run variable cost (TR < SVC)
Price < short run average variable cost (P< SAVC).

Shutdown Price:

The shutdown price is the price below which the firm will choose to produce no output in the short-run. It is equal to minimum average variable costs.

The firm will still produce if P < SAC, as long as it can cover its fixed costs. However, if price is less than the shutdown price, the firm will have smaller losses if it shuts down.

Shut down price = min of the average variable cost



Shut down price occurs when MC = AVC

Example

A perfectly competitive firm has the total cost curve: $TC = 2q^2 + 5q + 30$. What is the shut down price?

At shut down price : MC = AVC

$VC = 2q^2 + 5q$

$AVC = 2q + 5$

$MC = \frac{\partial TC}{\partial q} = 4q + 5$

MC = AVC $\rightarrow$ $2q + 5 = 4q + 5$ $\rightarrow$ $2q = 0$ $\rightarrow$ $q = 0$

AVC = 2q+ 5 ⇨ when q =0 ⇨ shut down price = $5

Example

If the cost function for John's Shoe Repair is $STC = 100 + 10q - q2 + ⅓q^3$, calculate the quantity at which the firm decides to shutdown in the short run.

At shut down price : MC = AVC

$VC = 10q - q^2 + ⅓q^3$ ⇨ $AVC = 10 - q + ⅓q^2$

$MC = \frac{\partial TC}{\partial q} = 10 - 2q + q^2$

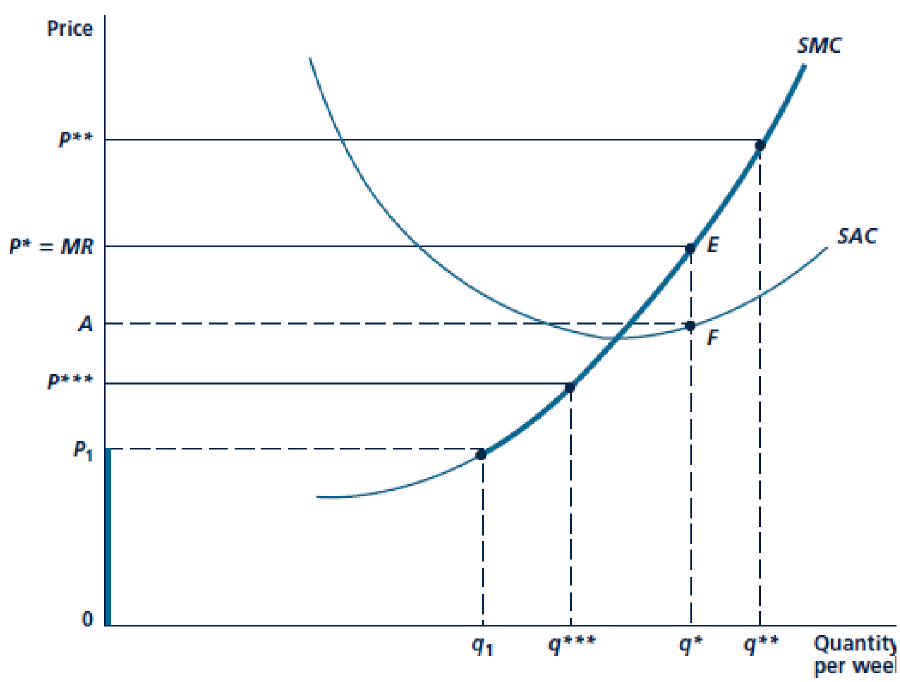
MC = AVC $\rightarrow$ $10 - 2q + q^2 = 10 - q + ⅓q^2$ $\rightarrow -q + \frac{2}{3}q^2 = 0$ $\rightarrow q(-1 + \frac{2}{3}q) = 0$

$q = 0$ or $-1 + \frac{2}{3}q = 0$ $\rightarrow \frac{2}{3}q = 1$ $\rightarrow q = \frac{3}{2}$

The Firm's Short-Run Supply Curve

The relationship between price and quantity supplied by a firm in the short run.

The positively sloped portion of the short-run marginal cost curve is the firm's short-run supply curve for this price-taking firm. That is, the curve shows how much the firm will produce for every possible market price. At a higher price of P**, for example, the firm will produce q** because it will find it in its interest to incur the higher marginal costs q** entails. With a price of P***, on the other hand, the firm opts to produce less (q***) because only a lower output level will result in lower marginal costs to meet this lower price. By considering all possible prices that



The firm's supply curve is the portion of the marginal cost curve for which marginal cost is greater than the average variable cost.

Example
A perfectly competitive firm has the total cost curve: $TC = 2q^2 + 5q + 30$.

a. What is the supply curve of the firm?

Firm's short-run supply curve is the positively sloped portion of the short-run marginal cost curve above the average variable cost curve.

$VC = 2q^2 + 5q$ ⇨ $AVC = 2q + 5$ and $MC = 4q + 5$

Supply curve: P = MC ⇨ $P = 4q + 5$

b. How many units of output will it produce at a market price of $65?

$P = 4q + 5$ ⇨ $65 = 4q + 5$ ⇨ $4q = 60$ ⇨ $q = 15$

Example
A competitive firm has the following short run cost function: $TC = q^3 - 8q^2 + 30q + 5$.

a. At what range of prices will the firm supply zero of output?

Competitive firms will shut down (supply zero output) if price is below average variable cost.
The firm will shut down when P< min AVC

At shut down price : MC = AVC

$VC = q^3 - 8q^2 + 30q \rightarrow AVC = \frac{VC}{q} = q^2 - 8q + 30$

$MC = \frac{\partial TC}{\partial q} = 3q^2 - 16q + 30$

$MC = AVC \rightarrow 3q^2 - 16q + 30 = q^2 - 8q + 30 \rightarrow 2q^2 - 8q = 0 \rightarrow q - 4 = 0 \rightarrow q = 4$

$AVC = q^2 - 8q + 30 = (4)^2 - 8(4) + 30 = 14$

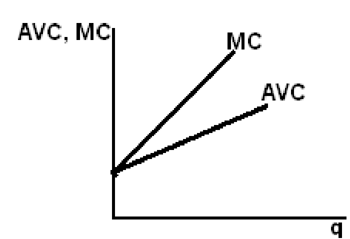
$the\ \ firm\ will\ shut\ down\ and\ supply\ zero\ of\ output\ \ when\ p \leq 14$

b. What is the short run supply curve?

Firm's short-run supply curve is the positively sloped portion of the short-run marginal cost curve above the average variable cost curve.

$VC = q^3 - 8q^2 + 30q$ ⇨ $AVC = \frac{VC}{q} = q^2 - 8q + 30$
$MC = 3q^2 - 16q + 30$

Supply curve: P = MC ⇨ $P = 3q^2 - 16q + 30$



c. At what price would the firm supply exactly 6 units of output?

Supply curve: $P = 3q^2 - 16q + 30$

$P = 3(6)^2 - 16(6) + 30 = 3x36 - 16x6 + 30 = 108 - 96 + 30 = 42$